Please check the examination details below before entering your candidate information

Candidate surname | Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Monday 15 June 2020

Afternoon (Time: 2 hours) | Paper Reference **4PM1/01**

Further Pure Mathematics
Paper 1

Calculators may be used. | Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

Geometric series

Sum to n terms, $S_n = \dfrac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \dfrac{a}{1 - r} \quad |r| < 1$

Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule
In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B \qquad \sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B \qquad \cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \dfrac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1   Differentiate with respect to x

$$6e^{3x^2}\cos 2x$$

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(Total for Question 1 is 3 marks)

P 6 2 2 8 0 A 0 3 3 2

2 (a) Using the axes below sketch the line with equation

(i) $y = 6$ (ii) $y + x = 10$ (iii) $y = 2x - 5$
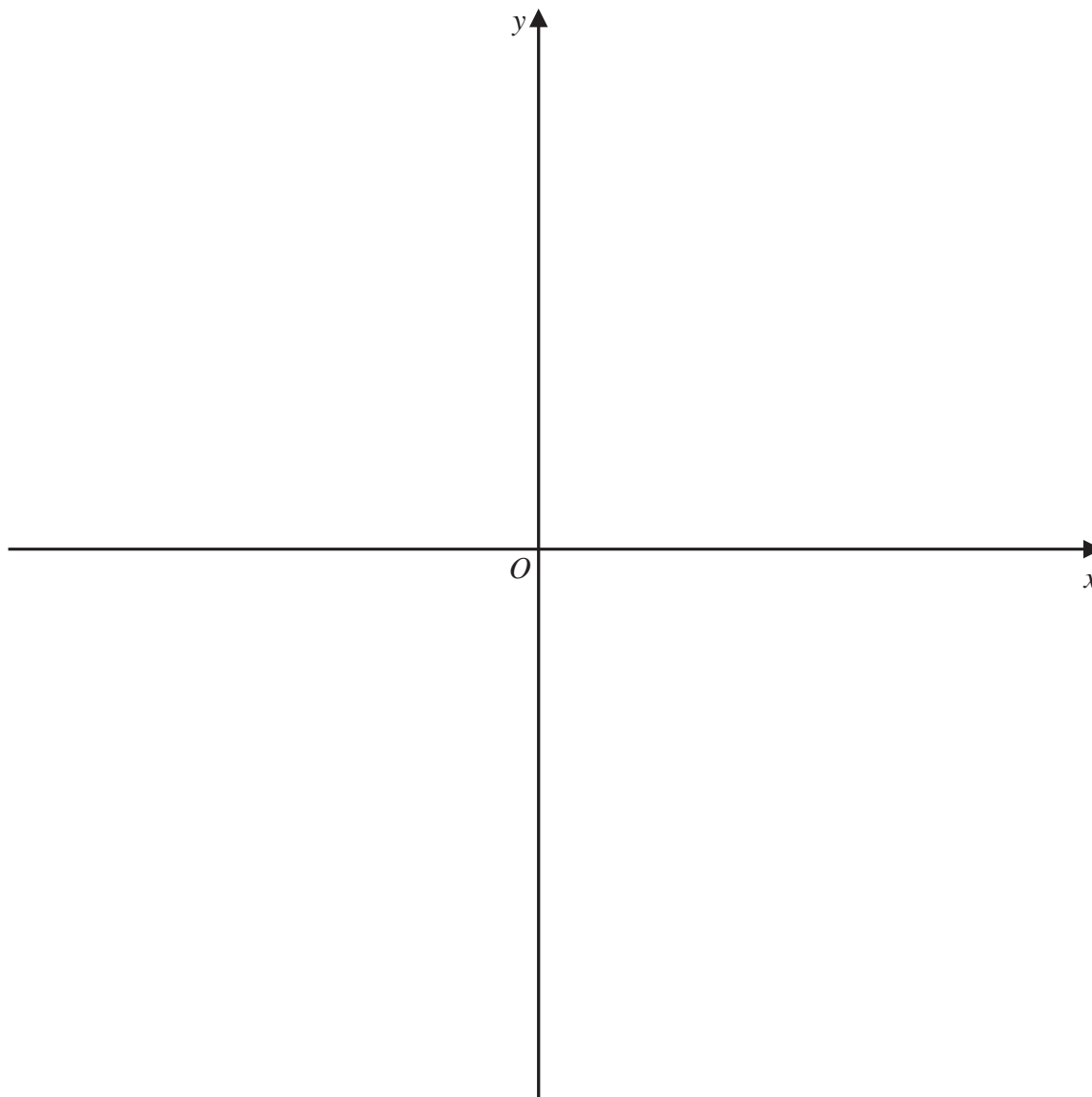
Show the coordinates of any point where each line crosses the coordinate axes.

(3)

(b) Show, by shading on your sketch, the region R defined by the inequalities

$y \leqslant 6$ $y + x \leqslant 10$ $y \geqslant 2x - 5$ $x \geqslant 0$

(1)

Question 2 continued

(Total for Question 2 is 4 marks)

P 6 2 2 8 0 A 0 5 3 2

3

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the right pyramid *ABCDE*. The base, *ABCD*, of the pyramid is a horizontal rectangle with $AD = 12$ cm and $CD = 16$ cm. The height *ME* of the pyramid is 14 cm where *M* is the point of intersection of the diagonals of the base.

The sloping edges, *EA*, *EB*, *EC* and *ED* of the pyramid are all of equal length.

(a) Calculate, to 3 significant figures, the length of a sloping edge.

(3)

Calculate, in degrees to one decimal place, the size of

(b) the angle between *AE* and the base,

(3)

(c) the angle between the plane *AED* and the base.

(3)

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Question 3 continued

(Total for Question 3 is 9 marks)

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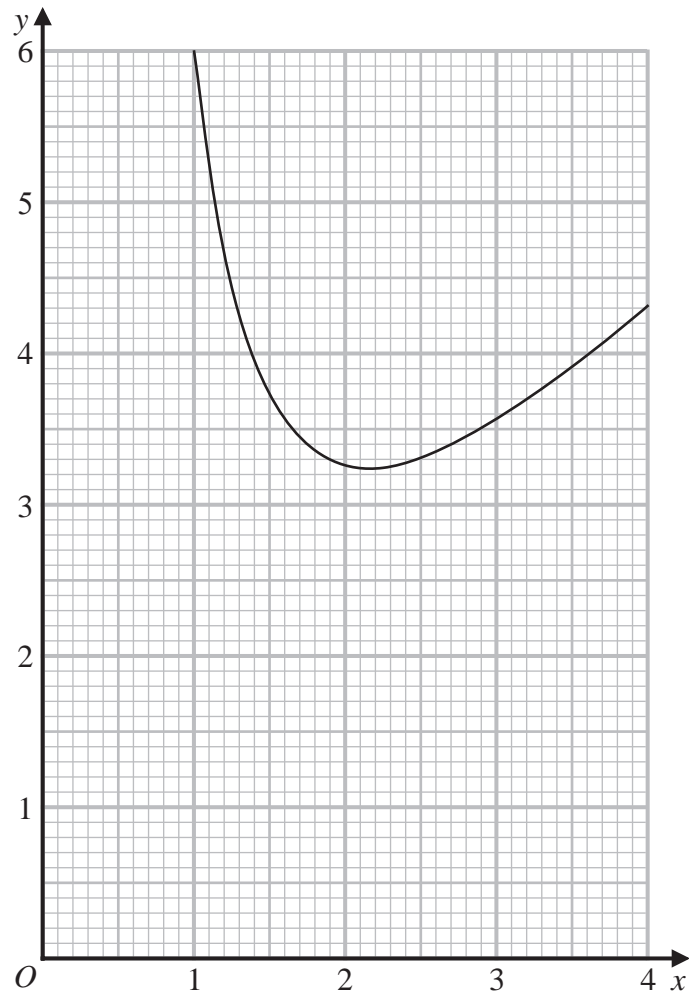
4

Figure 2

Figure 2 shows the graph of $y = x + \dfrac{5}{x^2}$ for $1 \leqslant x \leqslant 4$ drawn on a grid.

(a) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, for the roots of the equation

$$x^3 - 4x^2 + 5 = 0$$

in the interval $1 \leqslant x \leqslant 4$

(3)

(b) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, for the root of the equation

$$x^3 - x^2 - 5 = 0$$

in the interval $1 \leqslant x \leqslant 4$

(4)

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Question 4 continued

(Total for Question 4 is 7 marks)

5 The points P, Q, R and S have coordinates $(4, 7)$, $(3, 0)$, $(10, 1)$ and $(11, 8)$ respectively.

(a) Show, by calculation, that the lines PR and QS are perpendicular.

(3)

(b) Find the exact lengths of   (i) PR (ii) QS

(2)

(c) Find the area of the quadrilateral $PQRS$

(2)

Question 5 continued

(Total for Question 5 is 7 marks)

6 An arithmetic series A has first term a and common difference d.

The sum S_n of the first n terms of A is given by $S_n = n(15 + 2n)$

(a) Find the value of a and the value of d.

<div align="right">

(4)

</div>

(b) Find the 20th term of A.

<div align="right">

(2)

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Given that $S_{2p} - 2S_p = 1 + S_{(p-1)}$

(c) find the value of p.

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(4)

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Question 6 continued

(Total for Question 6 is 10 marks)

Turn over ▶

7

$$f(x) = x^2 - 9x + 14$$

Given that f(x) can be written in the form $(x + a)^2 + b$, where a and b are constants,

(a) find the value of a and the value of b.

(2)

(b) Hence, or otherwise, find

   (i) the minimum value of f(x)

   (ii) the value of x for which this minimum occurs.

(2)

The curve C has equation $y = $ f(x)

The line l has equation $y = x + 5$

(c) Use algebra to find the coordinates of the points of intersection of C and l.

(4)

(d) Use algebraic integration to find the exact area of the finite region bounded by C and l.

(5)

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Question 7 continued

P 6 2 2 8 0 A 0 1 5 3 2

Question 7 continued

P 6 2 2 8 0 A 0 1 6 3 2

Question 7 continued

(Total for Question 7 is 13 marks)

P 6 2 2 8 0 A 0 1 7 3 2

8 Given that $2xy + 5y = e^x$

(a) show that $\dfrac{dy}{dx} = \dfrac{y(2x + 3)}{2x + 5}$

(5)

(b) find the value of $\dfrac{dy}{dx}$ when $x = 0$

(2)

(c) find an equation of the normal to the curve with equation $2xy + 5y = e^x$ at the point where $x = 0$

Give your answer in the form $px + qy + r = 0$ where p, q and r are integers.

(3)

Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

P 6 2 2 8 0 A 0 2 1 3 2

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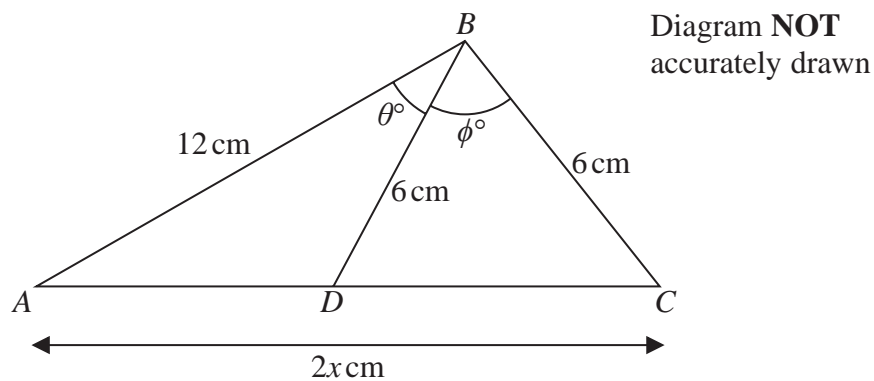
Figure 3

Figure 3 shows triangle ABC with $AB = 12$ cm, $BC = 6$ cm and $AC = 2x$ cm.

The point D is the midpoint of AC and $BD = 6$ cm.

$\angle ABD = \theta°$ and $\angle DBC = \phi°$ where $\theta \neq 0$ and $\phi \neq 0$

(a) Show that $\cos ADB = \dfrac{x^2 - 108}{12x}$

(2)

(b) Hence, or otherwise, show that $AC = 6\sqrt{6}$ cm.

(4)

(c) Show that $\sin(\theta° + \phi°) = \sin\phi°$

(4)

(d) Hence show that $\theta = 180 - 2\phi$

(2)

Question 9 continued

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Question 9 continued

Question 9 continued

(Total for Question 9 is 12 marks)

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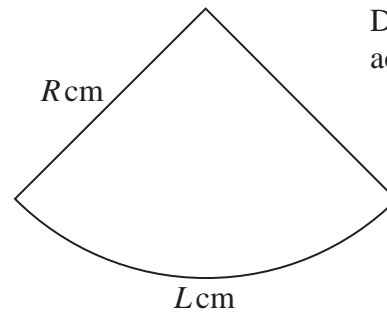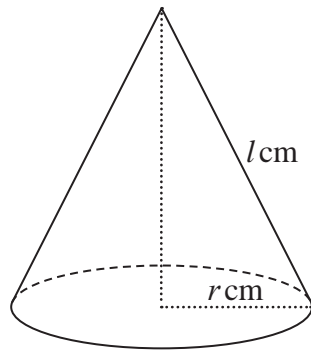
Figure 4

Figure 4 shows a right circular cone with base radius r cm and slant height l cm.
Figure 4 also shows a sector of a circle with radius R cm and arc length L cm.

The area of the curved surface of the cone is A cm^2

By considering how the sector of the circle can be folded to exactly form the curved surface of the cone with R and L suitably chosen,

(a) prove that $A = \pi r l$

(4)

Sand is poured onto a horizontal surface at a constant rate of 1.5 cm^3/s.
The sand forms a pile in the shape of a right circular cone with its base on the surface.
The curved surface area of the cone, A cm^2, increases in such a way that the height of the cone is always three times the radius of the base of the cone.

Given that $\dfrac{\mathrm{d}A}{\mathrm{d}r} = k\pi r$, where k is a constant,

(b) find the exact value of k.

(3)

(c) Calculate the rate, in cm^2/s, to 3 significant figures, at which the curved surface area of the pile is increasing when the height of the pile is 24 cm.

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Question 10 continued

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Question 10 continued

Question 10 continued

11

Diagram **NOT** accurately drawn

Figure 5

In Figure 5, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

The point C divides OB in the ratio $1:3$
The point D is the midpoint of AC

(a) Find, as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

   (i) $\overrightarrow{AC}$ (ii) $\overrightarrow{OD}$ (iii) $\overrightarrow{BD}$

(5)

The point E is such that $\overrightarrow{OE} = \lambda\overrightarrow{OA}$

Given that E, D and B are collinear

(b) find the value of λ

(4)

Given that $\dfrac{\text{area } \Delta OAC}{\text{area } \Delta OEB} = \mu$

(c) find the value of μ

(4)

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Question 11 continued

Question 11 continued

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(Total for Question 11 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS

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